ERRATUM TO
“RULED LINEAR WEINGARTEN SURFACES IN MINKOWSKI 3-SPACE”

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“Case 2” of the proof of the theorem in [1] has been corrected. We assume that \( a, b \in \mathbb{R} \) with \( 2a - b \neq 0 \).

Case 2: \( \beta' \) is null everywhere. We may assume without loss of generality that \( M \) can be parametrized by \( x(s,t) = \alpha(s) + t\beta(s) \), where \( \langle \beta,\beta \rangle = 1 \), \( \langle \beta',\beta' \rangle = 0 \), \( \langle \alpha',\alpha' \rangle = \xi \) \((= \pm 1)\), and \( F = \langle \alpha',\beta \rangle = 0 \). Since \( \beta \times \beta' \) and \( \beta' \) are orthogonal null vectors, they are collinear. We can assume further that \( \beta \times \beta' = \beta' \). Note that \( \beta' \) is a null direction in the hyperboloid of one sheet \( \{ \nu | \langle \nu,\nu \rangle = 1 \} \), so \( \beta \) is a straight line.

As usual, let \( Q := \langle \alpha'\beta' \rangle \); then \( Q = \langle \alpha',\beta' \rangle \). The components of the first fundamental form are \( E = \xi + 2Qt \), \( F = 0 \), \( G = 1 \). Thus \( EG - F^2 = \xi + 2Qt \).

Let \( P := \langle \alpha',\beta'' \rangle \); then \( \langle \alpha'',\beta' \rangle = Q - P \). By the moving frame \( \alpha',\beta,\alpha' \times \beta \) with signs \( \xi, +1, -\xi \), we get

\[
\beta'' = \xi P\alpha' - \xi P\alpha' \times \beta \quad \text{and} \quad \alpha'' = -Q\beta - \xi(\alpha''\alpha'\beta')\alpha' \times \beta.
\]

Since \( Q' = \langle \alpha'',\beta' \rangle + \langle \alpha',\beta'' \rangle = -\xi Q(\alpha''\alpha'\beta') + P \), it follows that \( \langle \alpha''\alpha'\beta' \rangle = \xi(P - Q')/Q \). The unit normal vector field is

\[
U = \frac{\alpha' \times \beta - t\beta'}{\sqrt{|\xi + 2Qt|}}
\]
This leads to the components of the second fundamental form
\[
e = \frac{\xi(P - Q') + tQ(2P - Q')}{Q \sqrt{\xi + 2Qt}}, \quad f = \frac{Q}{\sqrt{\xi + 2Qt}}, \quad \text{and} \quad g = 0.
\]
We have
\[
K = \frac{Q^2}{\xi + 2Qt}, \quad K_{II} = \frac{\xi Q'}{2Q \sqrt{\xi + 2Qt}^{3/2}} \quad \text{and} \quad H = \frac{\xi(P - Q') + tQ(2P - Q')}{2Q \sqrt{\xi + 2Qt}^{3/2}}.
\]
Since \(aK_{II} + bH + cK\) is constant along each ruling of the surface \(M\), \((aK_{II} + bH + cK)_t = 0\). This implies that \(c = 0\) and that
\[
\begin{align*}
(3a - 2b)Q' + bP &= 0, \quad (1) \\
(6a - 5b)Q' + 4bP &= 0, \quad \text{and} \quad (2) \\
bQ' - 2bP &= 0, \quad (3)
\end{align*}
\]
Since \(2a - b \neq 0\), we infer that \(Q' = 0\). Consequently, \(K_{II} = 0\). Suppose from now on that \(b \neq 0\). Hence \(Q' = 2P\), so \(P = Q' = 0\). Therefore, \(K_{II} = H = 0\) and \(M\) is a conjugate of Enneper’s surface of the 2nd kind.

Here is a revised statement of the main result:

**Theorem.** Let \(M : x(s, t) = \alpha(s) + t\beta(s)\) be a non-developable ruled surface in \(\mathbb{E}^3\) with non-null rulings. Then:

1. \(H = 0 \implies K_{II} = 0\). (The converse is true whenever \(\beta'\) is nowhere null.)
2. \(K_{II} = H \implies K_{II} = H = 0\).
3. Let \(a, b\) and \(c\) be real numbers such that \(aK_{II} + bH + cK\) is constant along each ruling of \(M\); then \(c = 0\). Moreover,
   \begin{enumerate}
   \item[(3.1)] if \(\beta'\) is nowhere null and \(a^2 + b^2 \neq 0\), then \(M\) is one of the right conoids in Examples 3.5 \& 3.7;
   \item[(3.2)] if \(\beta'\) is nowhere null and \(2a - b \neq 0\), then \(M\) is either a helicoid of the 1st kind, a helicoid of the 2nd kind, or a helicoid of the 3rd kind; and
   \item[(3.3)] if \(\beta'\) is null everywhere and \(2a - b \neq 0\), then \(M\) is \(II\)-flat; in this case, \(M\) is a conjugate of Enneper’s surface of the 2nd kind provided that \(b \neq 0\).
   \end{enumerate}
In each case, we eventually get \(aK_{II} + bH + cK = 0\).
References


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