FUZZY PRIME IDEAL OF A GAMMA-NEAR-RING

BY

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Abstract. In this paper, we introduce fuzzy prime ideal in Γ-near-rings and prove that if $\mu$ is a fuzzy prime ideal of a Γ-near-ring $M$ then $M_\mu = \{x \in M | \mu(x) = \mu(0)\}$ is a prime ideal of $M$, and conversely if $I$ is a prime ideal of $M$, then the fuzzy ideal defined by $\mu(x) = 1$ if $x \in I$ and $\mu(x) = s$ if $x \not\in I$ where $0 \leq s < 1$, is a fuzzy prime ideal of $M$. As a consequence, we obtain that an ideal $I$ of a Γ-near-ring $M$ is a prime ideal of $M$ if and only if its characteristic functions is a fuzzy prime ideal of $M$.

Introduction

A non-empty set $N$ with two binary operations “+” and “.” is called a near-ring if it satisfies the following axioms:

(i) $(N, +)$ is a group (not necessarily abelian),
(ii) $(N, \cdot)$ is a semigroup,
(iii) $(a + b) \cdot c = a \cdot c + b \cdot c$ for all $a, b, c \in N$.

Precisely speaking, it is a right near-ring because it satisfies the right distributive law. We denote $ac$ instead of $a \cdot c$. Moreover, a near-ring $N$ is said to be zero-symmetric if $n0 = 0$ for all $n \in N$, where 0 is the additive identity in $N$.

The notion of a Γ-near-ring, a concept more general than a near-ring and a Γ-ring, was defined by Satyanarayana [11]. Later several authors like Booth, Groenewald and Satyanarayana studied the ideal theory in Γ-near-rings (see [2], [3], [4], [5], [12] and [13]).

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Let \((M,+)\) be a group (not necessarily abelian) and \(\Gamma\) be a non-empty set. Then \(M\) is said to be a \(\Gamma\)-near-ring if there exists a mapping \(M \times \Gamma \times M \to M\) (the image of \((a,\alpha, b)\) is denoted by \(aab\)) satisfying the following conditions:

(i) \((a+b)\alpha c = a\alpha c + b\alpha c\),

(ii) \((aab)\beta c = a\alpha(b\beta c)\) for all \(a, b, c \in M\) and \(\alpha, \beta \in \Gamma\). Moreover, a \(\Gamma\)-near-ring \(M\) is said to be zero-symmetric if \(aa0 = 0\) for all \(a \in M\) and \(\alpha \in \Gamma\), where 0 is the additive identity in \(M\).

Let \(M\) be a \(\Gamma\)-near-ring. A normal divisor \((I,+)) of \((M,+))\) is called

(i) a left ideal if \(a(b+i) = ab\) for all \(a, b \in M\), \(\alpha \in \Gamma\), \(i \in I\),

(ii) a right ideal if \(ia\epsilon I\) for all \(a \in M\), \(\alpha \in \Gamma\), \(i \in I\),

(iii) an ideal if it is both a left ideal and a right ideal.

For any two subsets \(X\) and \(Y\) of a \(\Gamma\)-near-ring \(M\), the set \(\{x\alpha y \mid x \in X, \alpha \in \Gamma, y \in Y\}\) is denoted by either \(XY\) or \(YX\). An ideal \(I\) of a \(\Gamma\)-near-ring \(M\) is said to be prime if it satisfies the condition: for any ideals \(A, B \subseteq M\), \(A\Gamma B \subseteq I\) imply \(A \subseteq I\) or \(B \subseteq I\).

It is clear that if \(M\) is a \(\Gamma\)-near-ring then the elements of \(\Gamma\) acts as binary operations on \(M\) such that the system \((M,+,\gamma)\) is a near-ring for all \(\gamma \in \Gamma\). The relations between the concepts \(\Gamma\)-near-ring and near-ring were studied by Satyanarayana [13].

Throughout this paper, \(M\) stands for a zero-symmetric \(\Gamma\)-near-ring. The ideal generated by an element \(a \in M\) is denoted by \(\langle a \rangle\). For other definitions and preliminary results on \(\Gamma\)-near-rings, we refer [8, 12, 13].

The concept of a fuzzy subset of a non-empty set was introduced by Zadeh [16]. A fuzzy set in a set \(A\) is a function \(\mu : A \to [0, 1]\). For any \(t \in [0, 1]\), the set \(\mu_t = \{x \in A \mid \mu(x) \geq t\}\) is called a level subset of \(\mu\). For any two fuzzy sets \(\mu, \sigma\) in \(A\), we write \(\mu \subseteq \sigma\) if \(\mu(x) \leq \sigma(x)\) for all \(x \in A\). Let \(X\) and \(Y\) be any two sets, \(f : X \to Y\) be any function, \(\mu\) be any fuzzy subset of \(X\) and \(\sigma\) be any fuzzy subset of \(Y\).

A fuzzy subset \(f(\mu)\) of \(Y\) defined by

\[
(f(\mu))(y) = \begin{cases} 
\sup_{x \in f^{-1}(y)} \mu(x) & \text{if } f^{-1}(y) \neq \phi, y \in Y \\
0 & \text{if } f^{-1}(y) = \phi, y \in Y
\end{cases}
\]
is called the image of $\mu$ under $f$. A fuzzy subset $f^{-1}(\sigma)$ of $X$ defined by $(f^{-1}(\sigma))(x) = \sigma(f(x))$ for all $x \in X$, is called the pre-image of $\sigma$ under $f$.

Jun et al. [8] introduced the concept of a fuzzy ideal in $\Gamma$-near-rings and studied some fundamental properties.

A fuzzy set $\mu$ in $M$ is called a fuzzy left (resp. right) ideal of $M$ if it satisfies

(i) $\mu$ is a normal fuzzy divisor with respect to the addition, that is, for all $x, y \in M$, $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$, $\mu(y + x - y) \geq \mu(x)$,

(ii) $\mu(\alpha(x + v) - u\alpha v) \geq \mu(x)$ (resp. $\mu(xu) \geq \mu(x)$) for all $x, u, v \in M$ and $\alpha \in \Gamma$.

Note that if $\mu$ is both a fuzzy left ideal and a fuzzy right ideal, then $\mu$ is said to be a fuzzy ideal of $M$.

A fuzzy (left, right) ideal $\mu$ of $M$ with $\mu(0) = 1$ is called a normal fuzzy (left, right) ideal of $M$. Jun et al. [7] introduced the fuzzy maximal ideals and investigated some related properties. A fuzzy ideal $\mu$ of $M$ is said to be a fuzzy maximal ideal if it satisfies the two conditions: (i) $\mu$ is non-constant; and (ii) $\mu^*(x) = \mu(x) + 1 - \mu(0)$ for all $x \in M$.

Satyanarayana and Syam Prasad [14] introduced the concept of fuzzy cosets in $\Gamma$-near-rings and proved that the set of all fuzzy cosets forms a $\Gamma$-near-ring. It is also obtained three fundamental isomorphism theorems on the $\Gamma$-near-rings of fuzzy cosets.

For other preliminary definitions and results related to fuzzyness, we refer [8]. In this paper, we introduce fuzzy prime ideal in $\Gamma$-near-rings and prove that if $\mu$ is a fuzzy prime ideal of $M$, then $M_\mu = \{x \in M \mid \mu(x) = \mu(0)\}$ is a prime ideal of $M$, and conversely if $I$ is a prime ideal of $M$, then the fuzzy ideal defined by $\mu(x) = 1$ if $x \in I$ and $\mu(x) = s$ if $x \not\in I$ where $0 \leq s < 1$, is a fuzzy prime ideal of $M$. As a consequence, we obtain that an ideal $I$ of a $\Gamma$-near-ring is a prime ideal of $M$ if and only if its characteristic function is a fuzzy prime ideal of $M$.

1. Fuzzy Ideals

Now we state a fundamental result on fuzzy ideals. The proof is a straightforward verification.
Theorem 1.1. Let $\mu$ be a fuzzy ideal of $M$. Then

(i) $\mu(0) \geq \mu(x)$,
(ii) $\mu(x + y) = \mu(y + x)$,
(iii) $\mu(x - y) = \mu(0)$ implies $\mu(x) = \mu(y)$ for all $x, y \in M$.

Theorem 1.2. ([8, Theorem 3.5]) Let $\mu$ be a fuzzy set in $M$. Then $\mu$ is a fuzzy left (resp. right) ideal of $M$ if and only if each level subset $\mu_t$, $t \in \text{Im}(\mu)$, of $\mu$ is a left (resp. right) ideal of $M$.

In the proof of the following theorem, we use a notation, which is similar to the notation used by Reddy and Satyanarayana [10].

Theorem 1.3. If $\mu$ is a fuzzy ideal of $M$ and $a \in M$, then $\mu(x) \geq \mu(a)$ for all $x \in \langle a \rangle$.

Proof. By straightforward verification, we conclude that for $a \in M$, $\langle a \rangle = \bigcup_{i=0}^{\infty} A_i$ where $A_{k+1} = A_k^* \cup A_k^+ \cup A_k^0 \cup A_k^{++}$, $A_0 = \{a\}$ and

\[
\begin{align*}
A_k^* &= \{n + x - n \mid n \in M, x \in A_k\}, \\
A_k^+ &= \{n_1 \alpha(a + n_2) - n_1 \alpha n_2 \mid n_1, n_2 \in M, a \in A_k, \alpha \in \Gamma\}, \\
A_k^0 &= \{x - y \mid x, y \in A_k\}, \\
A_k^{++} &= \{x n \alpha \mid x \in A_k, \alpha \in \Gamma \text{ and } n \in M\}.
\end{align*}
\]

We prove that $\mu(u) \geq \mu(a)$ for all $\mu \in A_m$ ($m \geq 1$). For this, we use induction on $m$. It is obvious if $m = 0$. Suppose the induction hypothesis for $k$. That is, $\mu(x) \geq \mu(a)$ for all $x \in A_k$. Now let $v \in A_k^* \cup A_k^+ \cup A_k^0 \cup A_k^{++}$. Suppose $v \in A_k^*$. Then $v = n + x - n$ for some $x \in A_k$. Now $\mu(v) = \mu(n + x - n) \geq \mu(x)$ (since $\mu$ is a fuzzy ideal of $M$) $\geq \mu(a)$ (by induction hypothesis). Let $v \in A_k^0$. Then $v = x_1 - x_2$ for some $x_1, x_2 \in A_k$. Now $\mu(v) = \mu(x_1 - x_2) \geq \min\{\mu(x_1), \mu(x_2)\} \geq \mu(a)$, by induction hypothesis.

Suppose $v \in A_k^+$. Then $v = n_1 \alpha(x + n_2) - n_1 \alpha n_2$ for some $n_1, n_2 \in M$, $x \in A_k$ and $\alpha \in \Gamma$. Now $\mu(v) = \mu(n_1 \alpha(x + n_2) - n_1 \alpha n_2) \geq \mu(x)$ (since $\mu$ is a fuzzy ideal of $M$) $\geq \mu(a)$ (by induction hypothesis).

Suppose $v \in A_k^{++}$. Then $v = x n \alpha$ for some $x \in A_k$, $\alpha \in \Gamma$ and $n \in M$. Now $\mu(v) = \mu(x n \alpha) \geq \mu(x)$ (since $\mu$ is a fuzzy ideal of $M$) $\geq \mu(a)$ (by induction hypothesis).
hypothosis). Thus in all cases, we proved that \( \mu(v) \geq \mu(a) \) for all \( v \in A_{k+1} \). Hence by the principle of mathematical induction, we conclude that \( \mu(v) \geq \mu(a) \) for all \( v \in A_m \) and for all positive integers \( m \). Hence \( \mu(x) \geq \mu(a) \) for all \( x \in \langle a \rangle \).

**Corollary 1.4.** Let \( \mu \) be a fuzzy ideal of \( M \). If \( I = \langle a \rangle = \langle b \rangle \), then \( \mu(a) = \mu(b) \).

**Proof.** Since \( a \in \langle b \rangle \) and \( b \in \langle a \rangle \), we have \( \mu(a) \geq \mu(b) \) and \( \mu(b) \geq \mu(a) \), so \( \mu(a) = \mu(b) \).

2. Fuzzy Prime Ideal of a \( \Gamma \)-Near-Ring

**Definition 2.1.** A fuzzy ideal \( \mu \) of \( M \) is said to be a fuzzy prime ideal of \( M \) if \( \mu \) is not a constant function and for any two fuzzy ideals \( \sigma \) and \( \delta \) of \( M \), \( \sigma \circ \delta \subseteq \mu \) implies that either \( \sigma \subseteq \mu \) or \( \delta \subseteq \mu \).

**Theorem 2.2.** ([8, Theorems 3.2-3.3]) (i) Let \( \mu \) be a fuzzy left (resp. right) ideal of \( M \). Then the set \( M_\mu = \{ x \in M \mid \mu(x) = \mu(0) \} \) is a left (resp. right) ideal of \( M \).

(ii) Let \( A \) be a non-empty subset of \( M \) and \( \mu_A \) be a fuzzy set in \( M \) defined by

\[
\mu_A(x) = \begin{cases} 
    s, & \text{if } x \in A, \\
    t, & \text{otherwise},
\end{cases}
\]

for all \( x \in M \) and \( s, t \in [0, 1] \) with \( s > t \). Then \( \mu_A \) is a fuzzy left (resp. right) ideal of \( M \) if and only if \( A \) is a left (resp. right) ideal of \( M \). Moreover \( M_{\mu_A} = A \).

**Theorem 2.3.** If \( \mu \) is a fuzzy prime ideal of \( M \), then \( M_\mu = \{ x \in M \mid \mu(x) = \mu(0) \} \) is a prime ideal of \( M \).

**Proof.** By Theorem 2.2 (i), \( M_\mu \) is an ideal in \( M \).

To show that \( M_\mu \) is a prime ideal in \( M \), let \( A \) and \( B \) be two ideals of \( M \) such that \( AB \subseteq M_\mu \). Define the fuzzy subsets \( \sigma \) and \( \delta \) of \( M \) as

\[
\sigma(x) = \begin{cases} 
    \mu(0), & \text{if } x \in A, \\
    0, & \text{if } x \notin A,
\end{cases} \quad \delta(y) = \begin{cases} 
    \mu(0), & \text{if } y \in B, \\
    0, & \text{if } y \notin B.
\end{cases}
\]
By Theorem 2.2 (ii), we have the $\sigma$ and $\delta$ are fuzzy ideals.

Next we verify that $\sigma \circ \delta \subseteq \mu$. If $(\sigma \circ \delta)(x) = 0$ for all $x \in M$, then there is nothing to prove.

Since $(\sigma \circ \delta)(x) = \sup_{x=yz}\{\min\{\sigma(y), \delta(z)\}\}$, we have to consider only the cases where $x = yz$ and $\min\{\sigma(y), \delta(z)\} > 0$. For all these cases, $\sigma(y) = \delta(z) = \mu(0)$. So $y \in A$ and $z \in B$. Now $x = yz \in A \Gamma B \subseteq M_{\mu}$ implies that $\mu(x) = \mu(0)$. Hence $(\sigma \circ \delta)(x) \leq \mu(x)$ for all $x$ in $M$. Thus $\sigma \circ \delta \subseteq \mu$. Since $\mu$ is a fuzzy prime ideal, we have that $\sigma \subseteq \mu$ or $\delta \subseteq \mu$. Suppose $\sigma \subseteq \mu$. If $A \not\subseteq M_{\mu}$, then there exists $a \in A$ such that $a \not\in M_{\mu}$. This means $\mu(a) \neq \mu(0)$. Since $\mu$ is a fuzzy ideal, we have $\mu(0) \geq \mu(a)$. So $\mu(a) < \mu(0)$. Now $\sigma(a) = \mu(0) > \mu(a)$, a contradiction to the fact that $\sigma \subseteq \mu$. Thus we proved that if $\sigma \subseteq \mu$, then $A \subseteq M_{\mu}$. Similarly if $\delta \subseteq \mu$, then one can show that $B \subseteq M_{\mu}$. This shows that $M_{\mu}$ is a prime ideal in $M$.

**Definition 2.4.** Suppose $A$ and $B$ are sets such that $A \subseteq B$. Define $\chi_A : B \to [0, 1]$ by $\chi_A(x) = 1$ if $x \in A$ and $\chi_A(x) = 0$ if $x \not\in A$. Then $\chi_A$ is called as a characteristic function of $A$.

**Note 2.5.** If $I$ is an ideal of $M$, then by Theorem 2.2 (ii), $\chi_I : M \to [0, 1]$, the characteristic function of $I$, is a fuzzy ideal of $M$.

**Lemma 2.6.** If $\mu$ is a fuzzy prime ideal of $M$, then $\mu(0) = 1$.

**Proof.** Suppose $\mu$ is a fuzzy prime ideal of $M$. In a contrary way, suppose $\mu(0) < 1$. Since $\mu$ is not a constant, there exists $a \in M$ such that $\mu(a) < \mu(0)$.

Define the fuzzy subsets $\theta$ and $\sigma$ of $M$ as $\theta(x) = \mu(0)$ for all $x \in M$, and $\sigma(x) = \begin{cases} 1 & \text{if } \mu(x) = \mu(0) \\ 0 & \text{otherwise} \end{cases}$ for all $x \in M$. Since $\theta$ is a constant function, it is a fuzzy ideal. Note that $\sigma$ is the characteristic function of $M_{\mu}$. Now by Theorem 2.2 (ii), $\sigma$ is a fuzzy ideal of $M$.

Since $\sigma(0) = 1 > \mu(0)$ and $\theta(a) = \mu(0) > \mu(a)$, we have that $\sigma \not\subseteq \mu$, $\theta \not\subseteq \mu$. Let $b \in M$. We know that $(\sigma \circ \theta)(b) = \sup_{b=x \circ y}\{\min\{\sigma(x), \theta(y)\}\}$.

Now we prove that $\min\{\sigma(x), \theta(y)\} \leq \mu(b)$ where $b = x \circ y$. For this, we consider tow cases $\sigma(x) = 0$, and $\sigma(x) = 1$ in the following.
Case (i) Suppose $\sigma(x) = 0$. Then $\mu(x) < \mu(0)$ (by definition of $\sigma$). Now $\min\{\sigma(x), \theta(y)\} = \min\{0, \mu(0)\} \leq \mu(x\circ y) = \mu(b)$.

Case (ii) Suppose $\sigma(x) = 1$. Then $\mu(x) = \mu(0)$. Since $\mu$ is a fuzzy right ideal of $M$, we have that $\min\{\sigma(x), \theta(y)\} = \min\{1, \mu(0)\} = \mu(0) = \mu(x) \leq \mu(x\circ y) = \mu(b)$.

From the two cases, we conclude that $(\sigma^\circ \theta)(b) = \sup_{b=x\circ y}\{\min\{\sigma(x), \theta(y)\}\}$ $\leq \mu(b)$ and so $\sigma^\circ \theta \subseteq \mu$. Since $\mu$ is a fuzzy prime, we have that $\sigma \subseteq \mu$ or $\theta \subseteq \mu$. This is a contradiction. Therefore $\mu(0) = 1$.

**Theorem 2.7.** If $\mu$ is a fuzzy prime ideal of $M$, then $|\text{Im}(\mu)| = 2$. Moreover $\text{Im}(\mu) = \{1, s\}$ where $0 \leq s < 1$.

**Proof.** Suppose $\mu$ is a fuzzy prime ideal of $M$. We show that $\text{Im}(\mu)$ contains exactly two values. By Lemma 2.6, we know that $\mu(0) = 1$. Let $a, b$ be elements of $M$ such that $\mu(a) < 1$ and $\mu(b) < 1$. It is enough to show that $\mu(a) = \mu(b)$.

**Part (i)** Define the fuzzy subsets $\theta$ and $\sigma$ of $M$ as $\theta(x) = \mu(a)$ for all $x \in M$, and $\sigma(x) = \begin{cases} 1, & \text{if } x \in \langle a \rangle \\ 0, & \text{otherwise} \end{cases}$ for all $x \in M$. As in the proof of Lemma 2.6, we can conclude that $\theta$ and $\sigma$ are fuzzy ideals of $M$. Since $a \in \langle a \rangle$, we have that $\sigma(a) = 1 > \mu(a)$, and so $\sigma \not\subseteq \mu$. Let $z \in M$. We know that $(\sigma^\circ \theta)(z) = \sup_{z=x\circ y}\{\min\{\sigma(x), \theta(y)\}\}$. If $x \not\in \langle a \rangle$, then $\sigma(x) = 0$ and so $\min\{\sigma(x), \theta(y)\} = \min\{0, \mu(a)\} = 0 \leq \mu(x\circ y) = \mu(z)$. If $x \in \langle a \rangle$, then $\sigma(x) = 1$. Using Theorem 1.3, we have the $\min\{\sigma(x), \theta(y)\} = \min\{1, \mu(a)\} = \mu(a) \leq \mu(x)$. Since $\mu$ is a fuzzy right ideal of $M$, it follows that $\mu(a) \leq \mu(x) \leq \mu(x\circ y) = \mu(z)$. From these facts, we conclude that $\sigma^\circ \theta \subseteq \mu$. Since $\mu$ is a fuzzy prime ideal, we have that $\sigma \subseteq \mu$ or $\theta \subseteq \mu$.

Since $\sigma \not\subseteq \mu$, it follows that $\theta \subseteq \mu$. Now $\mu(b) \geq \theta(b) = \mu(a)$.

**Part (ii)** Now we construct fuzzy ideals $\delta$ and $\rho$ of $M$ as $\delta(x) = \mu(b)$ for all $x \in M$, and $\rho(x) = \begin{cases} 1, & \text{if } x \in \langle b \rangle \\ 0, & \text{otherwise} \end{cases}$ for all $x \in M$. As in part (i), we can verify that $\mu(a) \geq \mu(b)$.

Thus from the parts (i) and (ii), it follows that $\mu(a) = \mu(b)$.
Theorem 2.8. Let $I$ be an ideal of $M$ and $\mu$ be a fuzzy set in $M$ defined by

\[
\mu(x) = \begin{cases} 
1, & \text{if } x \in I \\
\sigma, & \text{otherwise}
\end{cases}
\]

for all $x \in M$ and $\sigma \in [0,1)$. If $I$ is a prime ideal of $M$, then $\mu$ is a fuzzy prime ideal of $M$.

Proof. Suppose $I$ is a prime ideal of $M$. Clearly $\mu$ is a non-constant and by Theorem 2.2 (ii), it is a fuzzy ideal of $M$.

Let $\sigma$ and $\delta$ be two fuzzy ideals of $M$ such that $\sigma \delta \subseteq \mu$, $\sigma \not\subseteq \mu$, $\delta \not\subseteq \mu$. Then $\sigma(x) > \mu(x)$ and $\delta(y) > \mu(y)$ for some $x, y \in M$. Now $\mu(x) \neq 1$ and $\mu(y) \neq 1$ and so $\mu(x) = \mu(y) = \sigma$, also $x, y \not\in I$. Since $I$ is a prime ideal, we have that $\langle x \rangle \Gamma \langle y \rangle \not\subseteq I$. Then $\mu(a) = \sigma$ and hence $(\sigma \delta)(a) \leq \mu(a) = \sigma$.

Suppose $a = cd$ where $c \in \langle x \rangle$, $d \in \langle y \rangle$ and $\alpha \in \Gamma$. Then

\[
s = \mu(a) \geq (\sigma \delta)(a) = \sup_{a=cd} \{\min\{\sigma(c), \delta(d)\}\}
\geq \min\{\sigma(c), \delta(d)\}
\geq \min\{\sigma(x), \delta(y)\} \quad \text{(by Theorem 1.3)}
\geq \min\{\mu(x), \mu(y)\} = \sigma.
\]

Therefore $(\sigma \delta)(a) > s$. This is a contradiction. Hence $\mu$ is a fuzzy prime ideal of $M$.

Corollary 2.9. Let $\mu$ be a fuzzy set in $M$ such that $\mu$ is two valued and $\mu(0) = 1$. If $M_\mu$ is a prime ideal of $M$, then $\mu$ is a fuzzy prime ideal of $M$.

Corollary 2.10. Let $I$ be an ideal of $M$ then $\chi_I$ is a fuzzy prime ideal of $M \Leftrightarrow I$ is a prime ideal of $M$.

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