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SOME INEQUALITIES FOR SUBMANIFOLDS IN LOCALLY CONFORMAL ALMOST COSYMPLECTIC MANIFOLDS

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Abstract. Motivated by [4], the inequalities for invariant submanifolds involving totally real sectional curvature and the scalar curvature are given. Moreover, the equality cases are also discussed.

1. Introduction

To find the relations between the main extrinsic invariants and the main intrinsic invariants of a submanifold is one of the basic interests in the submanifold theory. B. Y. Chen [3] gave a sharp inequality for a submanifold in a real space form involving the main intrinsic invariants (namely the sectional curvature and the scalar curvature) and the main extrinsic invariant (namely the squared mean curvature). Moreover, such inequality is extended in [5] and [8], respectively, to submanifolds tangent to the structure vector field ξ in cosymplectic space forms and those in locally conformal almost cosymplectic manifolds.

On the other hand, B. Y. Chen obtained the inequalities involving totally real sectional curvature and the scalar curvature for Kähler submanifolds in complex space forms (see [4]). Naturally, one expects same or similar results for submanifolds in locally conformal almost cosymplectic manifolds. Hence, to find the relations for invariant submanifolds involving totally real sectional curvature is expected which leads the authors to the present article.

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This paper is organised as follows: in Section 2, we introduce the necessary preliminaries for later discussion; the relations between totally real sectional curvature and the scalar curvature for invariant submanifolds in normal locally conformal almost cosymplectic manifolds are established in Section 3.

2. Preliminaries

Let $\widetilde{M}$ be a $(2m+1)$-dimensional manifold. We call $\widetilde{M}$ an almost contact manifold if the structure group $GL(2m+1)$ of its linear frame bundle is reducible to $U(m)\times\{1\}$ which is equivalent to the existence of a $(1,1)$ tensor field φ, a vector field ξ and a 1-form η satisfying

$$\varphi^2 = -I + \eta\otimes\xi,\ \ \eta(\xi) = 1, \tag{2.1}$$

where I is the identity endomorphism. Clearly, (2.1) gives

$$\varphi(\xi) = 0,\ \ \eta\circ\varphi = 0. \tag{2.2}$$

Since $U(m)\times\{1\}\subset O(2m+1)$, there exists a Riemannian metric g which satisfies

$$g(X,Y) = g(\varphi X,\varphi Y) + \eta(X)\eta(Y),\ \ \eta(X) = g(\xi,X), \tag{2.3}$$

for all $X,Y\in T\widetilde{M}$. In this case, we call $\widetilde{M}$ equipped with the structure (φ,ξ,η,g) an almost contact metric manifold. The almost contact structure is said to be normal if the induced almost complex structure J on the product manifold $\widetilde{M}\times\mathbb{R}$ defined by

$$J\left(X,\lambda\frac{d}{dt}\right) = \left(\varphi X-\lambda\xi,\eta(X)\frac{d}{dt}\right) \tag{2.4}$$

is integrable, where X is tangent to $\widetilde{M}$, t is the coordinate of $\mathbb{R}$ and λ is a smooth function on $\widetilde{M}\times\mathbb{R}$. The condition for (φ,ξ,η,g) being normal is equivalent to the vanishing of the torsion tensor

$$[\varphi,\varphi] + 2d\eta\otimes\xi, \tag{2.5}$$

where $[\varphi,\varphi]$ is the Nijenhuis tensor of φ.

If both the fundamental 2-form Φ and 1-form η are closed, where

$$\Phi(X,Y) = g(X,\varphi Y),\ \ X,Y\in T\widetilde{M},$$

then $\widetilde{M}$ is called an almost cosymplectic manifold. It is known that a normal almost cosymplectic manfold is cosymplectic ([1]). $\widetilde{M}$ is said to be a locally conformal almost cosymplectic manifold ([7]) if there exists a 1-form ω such that

$$d\Phi = 2\omega \wedge \Phi, \quad d\eta = \omega \wedge \eta, \quad d\omega = 0. \tag{2.6}$$

By [6], a structure (φ, ξ, η, g) to be normal locally conformal almost cosymplectic if and only if

$$(\widetilde{\nabla}_X \varphi)Y = f(g(\varphi X, Y)\xi - \eta(Y)\varphi X), \text{ for all } X, Y \in T\widetilde{M}, \tag{2.7}$$

where $\widetilde{\nabla}$ is the Levi-Civita connection of the Riemannian metric g and $\omega = f\eta$. It is easy to see from (2.7) that

$$\widetilde{\nabla}_X \xi = f(X - \eta(X)\xi). \tag{2.8}$$

A plane section $\pi \subset T_p\widetilde{M}$ is called totally real if $\varphi\pi$ is perpendicular to π and is denoted by π^r. A plane section $\pi \subset T_p\widetilde{M}$ is called a φ-section if it is spanned by X and φX, where X is a unit tangent vector in $T_p\widetilde{M}$ orthogonal to the structure vector field ξ. In this case, the sectional curvature $\widetilde{K}(X, \varphi X)$ is called the φ-sectional curvature of π and is denoted by $\widetilde{H}(X)$. The curvature tensor $\widetilde{R}$ of a normal locally conformal almost cosymplectic manifold $\widetilde{M}(m \geq 2)$ with pointwise constant φ-sectional curvature c is given by ([7])

$$\begin{aligned}\widetilde{R}(X,Y,Z,W) = {}& \frac{c-3f^2}{4}\{g(X,W)g(Y,Z) - g(X,Z)g(Y,W)\} \\ & + \frac{c+f^2}{4}\{g(X,\varphi W)g(Y,\varphi Z) - g(X,\varphi Z)g(Y,\varphi W) \\ & - 2g(X,\varphi Y)g(Z,\varphi W)\} \\ & - \left(\frac{c+f^2}{4} + f'\right)\{g(X,W)\eta(Y)\eta(Z) - g(X,Z)\eta(Y)\eta(W) \\ & + g(Y,Z)\eta(X)\eta(W) - g(Y,W)\eta(X)\eta(Z)\},\end{aligned} \tag{2.9}$$

where f is the function given by $\omega = f\eta$, and $f' = \xi(f)$.

Let M be an $(n+1)$-dimensional submanifold of $\widetilde{M}(c)$ and denote by h, ∇ and $\nabla^\perp$ the second fundamental form of M, the induced connections on M and the normal bundle $T^\perp M$. Then the Gauss and Weingarten formulas can be written as

$$\widetilde{\nabla}_X Y = \nabla_X Y + h(X,Y), \quad \widetilde{\nabla}_X N = -A_N X + \nabla^\perp_X N, \tag{2.10}$$

for all $X, Y \in TM$ and $N \in T^{\perp}M$, where A is the shape operator and is related to h by

$$g(h(X,Y),N) = g(A_N X, Y).$$

Let R be the Riemannian curvature tensor of M, then the Gauss equation is given by

$$R(X,Y,Z,W) = \widetilde{R}(X,Y,Z,W) + g(h(X,W),h(Y,Z)) - g(h(X,Z),h(Y,W)), \tag{2.11}$$

for all $X, Y, Z, W \in TM$.

Choose an orthonormal basis $\{e_1, \ldots, e_{n+1}\}$ of $T_pM,\ p \in M$, then the mean curvature vector $\sigma(p)$ and the scalar curvature $\tau(p)$ are given respectively by

$$\sigma(p) = \frac{1}{n+1}\sum_{i=1}^{n+1} h(e_i, e_i), \quad 2\tau(p) = \sum_{1 \le i \ne j \le n+1} K(e_i, e_j). \tag{2.12}$$

A submanifold M tangent to the structure vector field ξ is said to be an invariant submanifold of $\widetilde{M}$ if there exists an φ-invariant distribution $\mathcal{D}$ (that is, $\varphi(\mathcal{D}) = \mathcal{D}$) on M such that $TM = \mathcal{D} \oplus \mathbb{R} \cdot \xi$ (see [2]). And in what follows, we always assume that $\dim M = n+1$, $\dim \mathcal{D} = 2d$, where $n = 2d$.

3. Totally Real Sectional Curvature for Invariant Submanifolds

For the invariant submanifold $M(n \ge 2)$ of a normal locally conformal almost cosymplectic manifold $\widetilde{M}(c)$, we have orthogonal decomposition $T_pM = \mathcal{D}(p) \oplus \mathbb{R} \cdot \xi$ for each point $p \in M$. We choose an orthonormal basis $\{e_1, \ldots, e_d, e_{\overline{1}} = \varphi e_1, \ldots, e_{\overline{d}} = \varphi e_d, e_{2d+1} = \xi\}$ for T_pM and an orthonormal basis $\{\alpha_1, \ldots, \alpha_{m-d}, \alpha_{\overline{1}} = \varphi\alpha_1, \ldots, \alpha_{\overline{m-d}} = \varphi\alpha_{m-d}\}$ for $T_p^{\perp}M$. Then with respect to such an orthonormal basis, the $(1,1)$ tensor field φ on M is given by

$$\varphi = \begin{pmatrix} 0 & -I_d & 0 \\ I_d & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{3.1}$$

where I_d denotes an identity matrix of degree d. By (2.7) and (2.10), it is easy to see

$$\varphi h(X,Y) = h(\varphi X, Y), \quad h(\varphi X, \varphi Y) = -h(X,Y), \ \text{ for all } \ X, Y \in TM, \tag{3.2}$$

which implies that

$$A_{\varphi\alpha_p} = \varphi A_{\alpha_p} = -A_{\alpha_p}\varphi, \quad p \in \{1, \dots, m-d, \overline{1}, \dots, \overline{m-d}\}. \tag{3.3}$$

Therefore, the shape operators of M can be written as

$$A_{\alpha_p} = \begin{pmatrix} A'_{\alpha_p} & A''_{\alpha_p} & 0 \\ A''_{\alpha_p} & -A'_{\alpha_p} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad A_{\alpha_{\overline{p}}} = \begin{pmatrix} -A''_{\alpha_p} & A'_{\alpha_p} & 0 \\ A'_{\alpha_p} & A''_{\alpha_p} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad p \in \{1, \dots, m-d\}, \tag{3.4}$$

where A'_{α_p} and A''_{α_p} are $d \times d$ matrices. (3.4) means that M must be minimal.

For each $k \in \mathbb{R}$, $p \in M$, define an invariant $\delta_k^r(p)$ by

$$\delta_k^r(p) = \tau(p) - k \inf K^r(p), \tag{3.5}$$

where $\inf K^r(p) = \inf_{\pi^r} \{K(\pi^r)\}$ and π^r runs over all totally real plane sections in $\mathcal{D}(p) \subset T_pM$, then we have the following.

Theorem 3.1. *Let $M(n \geq 2)$ be an invariant submanifold in a normal locally conformal almost cosymplectic manifold $\widetilde{M}(c)$. Then*

$$\inf K^r(p) \leq \frac{c - 3f^2}{4}, \quad p \in M. \tag{3.6}$$

The equality in (3.6) holds at $p \in M$ if and only if p is a totally geodesic point.

Theorem 3.2. *Let $M(n \geq 4)$ be an invariant submanifold in a normal locally conformal almost cosymplectic manifold $\widetilde{M}(c)$.*

(1) *For each $k \in (-\infty, 4]$, $\delta_k^r(p)$ satisfies*

$$\delta_k^r(p) \leq \frac{n(n+1)(c-3f^2) + 3n(c+f^2)}{8} - n\left(\frac{c+f^2}{4} + f'\right) - k\frac{c-3f^2}{4}, \quad p \in M. \tag{3.7}$$

(2)

$$\delta_k^r(p) = \frac{n(n+1)(c-3f^2) + 3n(c+f^2)}{8} - n\left(\frac{c+f^2}{4} + f'\right) - k\frac{c-3f^2}{4}, \quad p \in M, \tag{3.8}$$

holds for some $k \in (-\infty, 4)$ if and only if p is a totally geodesic point.

(3)

$$\delta_4^r(p) = \frac{n(n+1)(c-3f^2)+3n(c+f^2)}{8} - n\left(\frac{c+f^2}{4}+f'\right) - (c-3f^2), \ \ p\in M, \tag{3.9}$$

if and only if there exists an orthonormal basis $\{e_1, \dots, e_d, e_{\overline{1}} = \varphi e_1, \dots, e_{\overline{d}} = \varphi e_d, e_{2d+1} = \xi\}$ *for* T_pM *and an orthonormal basis* $\{\alpha_1, \dots, \alpha_{m-d}, \alpha_{\overline{1}} = \varphi\alpha_1, \dots, \alpha_{\overline{m-d}} = \varphi\alpha_{m-d}\}$ *for* $T_p^{\perp}M$ *such that the shape operators of* M *take the following forms:*

$$A_{\alpha_p} = \begin{pmatrix} A'_{\alpha_p} & A''_{\alpha_p} & 0 \\ A''_{\alpha_p} & -A'_{\alpha_p} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad A_{\alpha_{\overline{p}}} = \begin{pmatrix} -A''_{\alpha_p} & A'_{\alpha_p} & 0 \\ A'_{\alpha_p} & A''_{\alpha_p} & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$A'_{\alpha_p} = \begin{pmatrix} h^p_{11} & h^p_{12} & 0 \\ h^p_{12} & -h^p_{11} & 0 \\ 0 & 0 & 0_{d-2} \end{pmatrix}, \qquad A''_{\alpha_p} = \begin{pmatrix} -h^{\overline{p}}_{11} & -h^{\overline{p}}_{12} & 0 \\ -h^{\overline{p}}_{12} & h^{\overline{p}}_{11} & 0 \\ 0 & 0 & 0_{d-2} \end{pmatrix}, \tag{3.10}$$

where $p \in \{1, \dots, m-d\}$.

To prove Theorem 3.1, we need the following.

Lemma 3.3. *Let* $M(n \geq 2)$ *be an invariant submanifold in a normal locally conformal almost cosymplectic manifold* $\widetilde{M}(c)$. *Then for all unit tangent vector fields* X, Y *orthogonal to the structure vector field* ξ *of* M, *we have*

$$K(X,Y) + K(X,\varphi Y) + \frac{3f^2}{2} = \frac{1}{4}\Big\{H(X+\varphi Y) + H(X-\varphi Y) + H(X+Y) + H(X-Y) - H(X) - H(Y)\Big\}, \tag{3.11}$$

as long as $g(X,Y) = g(X,\varphi Y) = 0$.

Proof. For each point $p \in M$, and any unit tangent vectors X and Y as stated in the lemma, we get from (2.9) and the Gauss equation (2.11) that

$$K(X,Y) + K(X,\varphi Y) = \frac{c-3f^2}{2} - 2 \parallel h(X,Y) \parallel^2, \tag{3.12}$$

$$H(X) + H(Y) = 2c - 2 \parallel h(X,X) \parallel^2 - 2 \parallel h(Y,Y) \parallel^2 . \tag{3.13}$$

(3.11) follows from (3.12) and (3.13), thus Lemma 3.3 is finished.

Proof of Theorem 3.1. For $p \in M$, define

$$\mathcal{D}^1(p) = \{X \in \mathcal{D}(p) \mid g(X,X) = 1\}, \tag{3.14}$$

then

$$U_p = \Big\{(X,Y) \mid X, Y \in \mathcal{D}^1(p) \text{ and } g(X,Y) = g(X,\varphi Y) = 0\Big\} \tag{3.15}$$

is a closed subset of $\mathcal{D}^1(p) \times \mathcal{D}^1(p)$. Obviously, if $\{X, Y\}$ spans a totally real plane section, then $\{X + \varphi Y, X - \varphi Y\}$ and $\{X + Y, X - Y\}$ both span totally real plane sections. Let

$$\hat{H}(X,Y) = H(X) + H(Y), \qquad (X,Y) \in U_p, \tag{3.16}$$

then there exists $(\overline{X}, \overline{Y}) \in U_p$, such that $\hat{H}(\overline{X}, \overline{Y})$ attains an absolute maximum value. From (3.11) it follows that

$$K(\overline{X}, \overline{Y}) + K(\overline{X}, \varphi\overline{Y}) + \frac{3f^2}{2} \leq \frac{1}{4}\hat{H}(\overline{X}, \overline{Y}). \tag{3.17}$$

In addition, for unit tangent vector $X \in \mathcal{D}^1(p)$, it is easy to see that

$$H(X) \leq c. \tag{3.18}$$

By (3.17) and (3.18) we obtain

$$K(\overline{X}, \overline{Y}) + K(\overline{X}, \varphi\overline{Y}) \leq \frac{c - 3f^2}{2}, \tag{3.19}$$

which implies (3.6).

If the equality in (3.6) holds at p, then

$$K(X,Y) + K(X, \varphi Y) \geq 2 \inf K^r = \frac{c - 3f^2}{2}, \tag{3.20}$$

which combined with (3.12) gives

$$h(X,Y) = 0, \tag{3.21}$$

for all $X, Y \in \mathcal{D}^1(p)$ with $g(X,Y) = g(X, \varphi Y) = 0$. It follows that

$$h(X + \varphi Y, X - \varphi Y) = 0, \;\; h(X + Y, X - Y) = 0, \tag{3.22}$$

that is $h(X,X)=0$. Since every tangent vector of $\mathcal{D}(p)$ must be lie in a totally real plane section of $\mathcal{D}(p)$, therefore,

$$h(X,Y)=0, \text{ for all } X,Y\in\mathcal{D}(p). \tag{3.23}$$

(3.23) and (2.8) show that p must be a totally geodesic point.

The converse is straightforward.

Proof of Theorem 3.2. Since an invariant submanifold $M(n\geq 2)$ of a normal locally conformal almost cosymplectic manifold $\widetilde{M}(c)$ is minimal, we see from the Gauss equation (2.11) that the scalar curvature τ and the second fundamental form h of M at p satisfy

$$2\tau(p)=\frac{n(n+1)(c-3f^2)+3n(c+f^2)}{4}-2n\left(\frac{c+f^2}{4}+f'\right)-\parallel h\parallel^2, \tag{3.24}$$

which implies

$$\tau(p)\leq\frac{n(n+1)(c-3f^2)+3n(c+f^2)}{8}-n\left(\frac{c+f^2}{4}+f'\right), \tag{3.25}$$

with the equality holding if and only if p is a totally geodesic point.

Choose an orthonormal basis $\{e_1,\ldots,e_d,e_{\overline{1}}=\varphi e_1,\ldots,e_{\overline{d}}=\varphi e_d,e_{2d+1}=\xi\}$ for T_pM and an orthonormal basis $\{\alpha_1,\ldots,\alpha_{m-d},\alpha_{\overline{1}}=\varphi\alpha_1,\ldots,\alpha_{\overline{m-d}}=\varphi\alpha_{m-d}\}$ for $T_p^{\perp}M$ such that a given totally real section π=Span$\{e_1,e_2\}$, $\pi\subset\mathcal{D}(p)$. By (3.4) and (3.24) we obtain

$$\begin{aligned}
&-2\tau(p)+\frac{n(n+1)(c-3f^2)+3n(c+f^2)}{4}-2n\left(\frac{c+f^2}{4}+f'\right)\\
&\geq 4\sum_{p=1}^{m-d}\left\{(h_{11}^p)^2+(h_{22}^p)^2+2(h_{12}^p)^2+(h_{11}^{\overline{p}})^2+(h_{22}^{\overline{p}})^2+2(h_{12}^{\overline{p}})^2\right\}\\
&\geq -8\sum_{p=1}^{m-d}\left\{h_{11}^p h_{22}^p-(h_{12}^p)^2+h_{11}^{\overline{p}}h_{22}^{\overline{p}}-(h_{12}^{\overline{p}})^2\right\}\\
&=-8\left(g(h(e_1,e_1),h(e_2,e_2))-\parallel h(e_1,e_2)\parallel^2\right)\\
&=-8\left(K(\pi)-\frac{c-3f^2}{4}\right).
\end{aligned} \tag{3.26}$$

(3.26) gives

$$\tau(p)-4K(\pi)\leq\frac{n(n+1)(c-3f^2)+3n(c+f^2)}{8}-n\left(\frac{c+f^2}{4}+f'\right)-(c-3f^2), \tag{3.27}$$

with equality holding if and only if

$$h^p_{11}+h^p_{22}=0,\ h^p_{1j}=h^p_{2j}=h^p_{ij}=0, p\in\{1,\dots,m-d,\overline{1},\dots,\overline{m-d}\},\ i,j\in\{3,\dots,n+1\}. \tag{3.28}$$

(1) Clearly, (3.27) yields

$$\begin{aligned}\delta^r_4(p)&=\tau(p)-4\inf K^r(p)\\&\leq\frac{n(n+1)(c-3f^2)+3n(c+f^2)}{8}-n\left(\frac{c+f^2}{4}+f'\right)-(c-3f^2).\end{aligned} \tag{3.29}$$

For $\widetilde{\lambda}\in(0,\infty)$, from (3.25) it follows that

$$\widetilde{\lambda}\tau(p)\leq\frac{\widetilde{\lambda}(n(n+1)(c-3f^2)+3n(c+f^2))}{8}-\widetilde{\lambda}n\left(\frac{c+f^2}{4}+f'\right), \tag{3.30}$$

which together with (3.29) show that the inequality (3.7) holds when $k\in(0,4)$. In particular, (3.25) and (3.29) are, respectively, special cases of $k=0$ and $k=4$. The inequality (3.7) with $k\in(-\infty,0)$ follows from (3.6) and (3.25).

(2) If equality (3.8) holds at p for some $k\in(-\infty,4)$, then we have the following three cases:

A. $k=0$, that is,

$$\delta^r_0(p)=\tau(p)=\frac{n(n+1)(c-3f^2)+3n(c+f^2)}{8}-n\left(\frac{c+f^2}{4}+f'\right), \tag{3.31}$$

then (3.25) implies that p is a totally geodesic point.

B. $k\in(0,4)$, then (3.29) and the definition of $\delta^r_k(p)$ yield

$$\begin{aligned}&\delta^r_k(p)=\tau(p)-k\inf K^r(p)=(1-\frac{k}{4})\tau(p)+\frac{k}{4}\delta^r_4(p)\\&\leq\frac{n(n+1)(c-3f^2)+3n(c+f^2)}{8}-n\left(\frac{c+f^2}{4}+f'\right)-k\frac{c-3f^2}{4},\end{aligned} \tag{3.32}$$

which means that the equality in (3.25) holds, so p is a totally geodesic point.

C. $k \in (-\infty, 0)$, then (3.25) together with (3.6) gives

$$\begin{aligned}
&\delta_k^r(p) = \tau(p) - k \inf K^r(p) \\
&\leq \frac{n(n+1)(c-3f^2)+3n(c+f^2)}{8} - n\left(\frac{c+f^2}{4} + f^{'}\right) - k\frac{c-3f^2}{4}.
\end{aligned} \tag{3.33}$$

Similarly, we can conclude that the equality in (3.25) holds. Hence, p must be a totally geodesic point.

Conversely, if p is a totally geodesic point, then (3.6) and (3.25) gives (3.8).

(3) From (3.9) we know that the inequality in (3.27) becomes equality, which yields (3.28). By this we conclude that the shape operator of M at p has components as in (3.10) with respect to some orthonormal basis

$$\{e_1, \dots, e_d, \varphi e_1, \dots, \varphi e_d, e_{2d+1} = \xi, \alpha_1, \dots, \alpha_{m-d}, \varphi\alpha_1, \dots, \varphi\alpha_{m-d}\} \tag{3.34}$$

for $T_p\widetilde{M}$.

Conversely, suppose that the shape operator of M at p has components as in (3.10) with respect to some orthonormal basis (3.34) for $T_p\widetilde{M}$, then the inequality in (3.27) becomes an equality, which together with (3.7) gives

$$\begin{aligned}
&\frac{n(n+1)(c-3f^2)+3n(c+f^2)}{8} - n\left(\frac{c+f^2}{4} + f^{'}\right) - (c-3f^2) \\
&\geq \delta_4^r(p) \geq \tau(p) - 4K(\pi) \\
&= \frac{n(n+1)(c-3f^2)+3n(c+f^2)}{8} - n\left(\frac{c+f^2}{4} + f^{'}\right) - (c-3f^2),
\end{aligned} \tag{3.35}$$

this yields (3.9).

The proof of Theorem 3.2 is finished.

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